RELIABILITY STUDIES ON THE INFLUENCE OF JOINT CLEARANCE ON THE KINEMATICS OF THE NOSE LANDING GEAR MECHANISM OF A TRANSPORT AIRCRAFT USING CONTACT THEORY

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ABSTRACT

Contact between two objects is an important facet in multibody dynamics. It is a discontinuous, non-linear phenomenon and consequently it requires iterative simulations. The paper presents the reliability evaluation of the retraction landing gear mechanism by three contact models Viz. Impact Function Model, Coefficient of Restitution Model and Clearance Link Model. The simulations have been performed using the standard commercial multibody dynamics software ADAMS. The precision of these simulations depends on user-defined parameters like stiffness, Damping, Penetration Depth, Force exponent, Penalty and Restitution Coefficient that impacts the overall reliability of the mechanism. The optimal value of these parameters have been obtained by an optimization process using Design of Experiments tool available in ADAMS to match with the nominal values without any clearance.. The overall reliability of the mechanism has been evaluated at different instants of the retraction cycle by using Response Surface Based Monte Carlo Simulation and Direct Monte Carlo Simulation by using in house codes created in MATLAB software. The comparison, significance and accuracy of the results obtained using the above -mentioned approaches has been discussed and the impact based contact modelling for the clearance appears to be accurate and realistic for practical applications.

Keywords: reliability, mechanism, design of experiments, Monte Carlo simulation, impact model.

INTRODUCTION

Landing gear is one of the critical subsystems of an aircraft. According to a study there are 1408 system related accidents between 1958 and 1993 in total; about one third (456) of these accidents were related to landing gears. This is more than twice as many as the next most failure prone category engines, which accounted for 192 accidents [1]. The objective of a landing gear in a transport aircraft is to function as a suspension system during landing, take-off and taxi; thereby regulating the loads being transmitted back to the airframe. After take-off the landing gear is retracted back to minimize the aerodynamic drag during its flight. Accurate extension and retraction of Landing Gear Mechanism is necessary for the safe landing of aircraft.

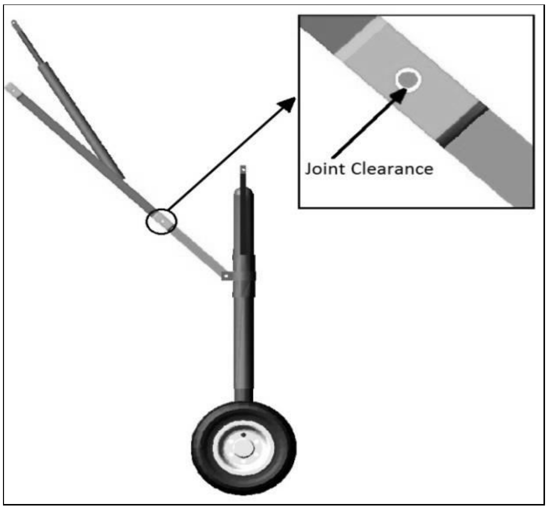

Figure-1. Geometry of nose landing gear.

The successful deployment of Landing gear Up-lock and Down-lock is dependent on the accuracy of the Landing Gear mechanism [2]. Any deviation at the extreme positions of extension and retraction would result in the incorrect release or jamming of the Up lock and down lock. The deviations may occur due to improper design, Manufacturing errors, Assembly defects or Operational wear and tear. The presence of significant clearances at the joints of the nose landing gear would most definitely induce higher wear and tear as a result the analysis of joint clearances and its impact on the reliability of the mechanism during the retraction and extension operations has been studied using the clearance model, impact and coefficient of restitution approach using the commercially available ADAMS software and in-house codes created in MATLAB.

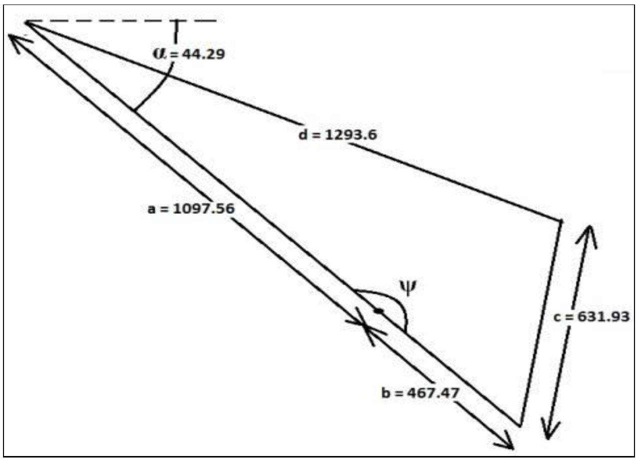

Figure-2. Schematic of the landing gear.

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The landing gear retraction-extension mechanism is a four bar mechanism with four links a, b, c and d, and driven by a hydraulic actuator. The landing gear considered in this study is as shown in Figure-1.Figure-2 shows the schematic of the landing gear four bar mechanism. When α is 44.29^0, the landing gear is fully extended and when α is 12.068^0 the landing gear is fully retracted. The Nose landing gear has been analysed using the Impact function model, Coefficient of Restitution and Clearance Link Model approach. The Clearance Link Model assumes a rigid link as the clearance in the revolute joint ignoring the realistic effects of Friction, Stiffness and damping. The Impact Function Model and the Restitution Model overcomes this drawback of the Clearance Link Model.

IMPACT MODEL

The IMPACT function model for contact modelling which is deduced from Hertzian contact theory: The Restoring Normal force (F) measured at the contact is as expressed below:

$$F = K_c(X_1 - X)$$
$$F = 2aE(X_1 - X) \tag{1}$$

$$F = 2\left(\frac{3L\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1}}{4\left(\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2}\right)^{-1}}\right)^{\frac{1}{3}}\left(\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2}\right)^{-1}(X_1 - X) \tag{2}$$

The spring force (F) depends on a stiffness parameter (K_c=2aE) and the penetration depth (x_1-x) . The stiffness further depends on both materials Young's Moduli (E) and Poisson's Ratios(v), both objects' radii (R_i) and the force with which the objects are pressed together. The IMPACT function uses a stiffness parameter that is related to the Hertzian contact stiffness; however, the load appears to vary with the penetration depth [3]. A greater penetration depth leads to a greater restoring normal force (F). Therefore the contact stiffness is not constant, making the force non-linear and due to this non-linearity, the IMPACT function does not only use a static stiffness parameter (K), but also an additional force exponent (e):

$$F = K_c\,(x_1 - x)^e \tag{3}$$

It should be noted that the value of the force exponent should be greater than 1, to increase the contact stiffness for increasing penetration depths. Hertzian contact theory states that at contact, both objects deform ever so slightly to create an elliptical contact area. Deformation dissipates energy from the system, so the IMPACT function has to take this dissipation into account and ADAMS uses a damping parameter to create a damping force that dissipates energy from the system. Since the dissipation of energy depends on the contact area and contact stiffness, the damping value in the IMPACT function is recommended to be a small fraction of the stiffness value, usually: $C_{max} < 0.01$ k [3].

The choice of IMPACT function model is thus followed by the input of four parameters Viz: stiffness, force exponent, damping and penetration depth. The IMPACT function in ADAMS has seven arguments, which can be expressed as:

$$\mathrm{IMPACT}(x, \dot{x}, x_1, K, e, C_{max}, d) \tag{4}$$

Where:

- x distance variable used to compute the IMPACT function.
- $\dot{x}$ time derivative of x to IMPACT.
- x_1 A positive real variable that specifies the free length of If x is less than x1, then Adams calculates a positive value for the force. Otherwise, the force value is zero.
- K A non-negative real variable that specifies the stiffness of the boundary surface interaction.
- e A positive real variable that specifies the exponent of the force deformation characteristic. For a stiffening spring characteristic, e > 1.0. For a softening spring characteristic, 0 < e < 1.0.
- C_{max} A non-negative real variable that specifies the maximum damping coefficient.
- d A positive real variable that specifies the boundary penetration at which Adams applies full damping.

The first three arguments are determined every time step of the simulation and are geometry-related expressions. The other four arguments are the user-specified parameters

$$F = \begin{cases} 0 & \text{If } X > X_1 \\ K_c(X_1 - X)^e - C_{max}\dot{X} * \mathrm{STEP}\,(X, X_1 - d, 1, X_1, 0) & \text{If } X \le X_1 \end{cases} \tag{5}$$

The force 'F' activates when the distance between the two objects is smaller than the free length of x. When the force becomes non-zero and consists of two parts: an exponential spring force and a damping force that follows a step function. It should be noted that both forces are strictly positive. The reason is that the calculated normal force should oppose the compression that occurs during penetration. Negative forces would support the compression, which a real normal force would never do.

As soon as, 'x' becomes smaller than x_1, a positive spring force is created, assuming that is positive as it is supposed to be. Unlike in a linear spring (F= -Kx), the spring force is exponential. For 0<e<1, the spring force concaves down and at x=0, the slope is infinite. For e=1, the spring force is linear, so at x=0, the slope has a finite value. For e>1, the spring force concaves up and at x=0, the slope is zero. It is recommended to use e>1, so that the slope of the spring force is continuous even when passing from the non-contact domain to the contact domain. From

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experience it can be said that hard metals require a value of e≅ 2.2, softer metals require a value of e ≅ 1.5 and softer materials like rubber require a value of e ≅ 1.1. From Hertzian contact theory follows that the stiffness of the contact, K, is based on both material properties (Young's Modulus and Poisson's Ratio) and geometrical properties (radius of curvature). Determining the value can be done by trial-and-error or by consulting experience of other users. Since the relative velocity will have a non-zero value when 'x' becomes smaller than 'x_1', a linear damper (F = -C$\dot{x}$) would induce a discontinuity in the damping force. To avoid this problem, a cubic step function is used to increase the damping force from zero to $C_{max}C_{max}\dot{x}$ within the penetration depth. It has to be noted that the penetration depth is not necessarily the maximum penetration depth during a collision and is merely a penetration depth at which the damping is at maximum. Figure 3.shows the behaviour of the IMPACT function's spring force and damping force.

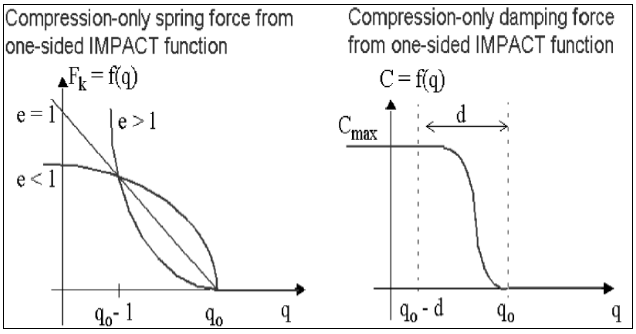

Figure-3. Plots for the two force components of the IMPACT function [3].

COEFFICIENT OF RESTITUTION MODEL

The coefficient of restitution model uses restitution coefficient (COR) defines a continuum between a perfectly elastic (COR = 1.0) and perfectly inelastic (COR = 0.0) collision [3].

The difference between both limits is that in an elastic collision the kinetic energy is conserved and in an inelastic collision the kinetic energy is not conserved. In a perfectly inelastic collision the reduction of kinetic energy equals the total kinetic energy before the collision in a centre-of-momentum frame. Even though the behaviour of kinetic energy differs in these cases, in all collisions the total momentum is conserved [3]. For simple collisions the object velocities can be calculated with the conservation of momentum and by definition of COR can be expressed as:

$$\mathrm{COR} = \frac{v_b - v_a}{u_a - u_b} \tag{6}$$

$$m_a u_a + m_b u_b = m_a v_a + m_b v_b \tag{7}$$

Where, m_a and m_b is the masses of the objects under contact. u_a and u_b are the initial velocities of the objects. From these equations the velocities $v_a$and v_b of the two objects can be derived if the values of COR is known.

$$v_a = \frac{m_a u_a + m_b u_b + m_b \mathrm{Cor}(u_b - u_a)}{m_a + m_b}$$
$$v_b = \frac{m_a u_a + m_b u_b + m_a \mathrm{Cor}(u_a - u_b)}{m_a + m_b} \tag{8}$$

Adams calculates the normal force, which requires the use of a penalty parameter, which is similar to a stiffness parameter. The disadvantage of using this so-called penalty regularization is that the user is responsible for setting an appropriate penalty parameter [3]. A small value will result in disobeying the impenetrability constraint (no negative gap between objects) and therefore inaccurate results. If the penalty parameter approaches infinity, the impenetrability constraint would be perfectly met. Integration difficulties will arise, though. In a MMKS (mm, kg ,N ,s ,deg) unit model, a value of $1e^5$ or $1e^6$ is appropriate.

The function for the normal force associated with the POISSON restitution model available in ADAMS is expressed as:

$$F = p(\varepsilon - 1)\dot{x} \tag{9}$$

Where, p is the penalty parameter, ε is COR and $\dot{x}$ is the time derivative of x, the clearance gap. The input parameters based on the restitution approach used has been given in the results section.

METHODOLOGY

Two methods have been used for the computation of the reliability of the mechanism for the variation in joint clearance in the Nose landing Gear. Viz; Response Surface Method based Monte Carlo and Direct Monte Carlo simulations. The Design Variable in the present study is the Joint Clearance (CL) = 1.5 mm

The output response is the angle (ψ) measured between the Upper drag Link (a) and Lower drag Link (b) at different retraction times. The Performance function is the measured deviation between (ψ) and (ψ1), where ψ1= angle measured between links a and b with no joint clearance. The Performance function can be thus stated as:

$$\left|\varphi_1 - \varphi\right| = \text{Deviation (dev)} \{ \text{If dev} > 1.85^0; \text{Failure} \tag{10}$$

The Response Surface Method (RSM) in Landing Gear Kinematic analysis has been used to determine the effects of Joint Clearance (CL) that could affect the angle ψ. The procedure adopted has been by using the design of experiments approach with three level full factorial designs by varying the Joint Clearance. For a three level, one variable full factorial design 3^1=3 runs have to be performed. This gives three output configurations for each angular step considered for the simulations. The response surface has been fitted accordingly from the output responses obtained for the three points. A second-order model has been found to be accurate enough in approximating a portion of the true response surface with parabolic curvature, which is expressed as:

$$y = \beta_0 + \sum_{j=1}^{q} \beta_j x_j + \sum_{i=1}^{q} \beta_{jj} x_j^2 + \sum \sum_{i<j} \beta_{ij} x_i x_j + \varepsilon$$
$$= \beta_0 + x'_i \beta + x'_i \beta x_i + \varepsilon_{ij},$$
$$\text{where } x_i = (x_{1i}, x_{2i}, \ldots, x_{iq})', \ \beta = (\beta_1, \beta_2, \ldots, \beta_q)'. \quad (11)$$

The second-order model is flexible, because it can take a variety of functional forms and approximates the response surface locally. Therefore, this model is usually a good estimation of the true response surface. The final step is to use the Monte Carlo Simulation technique to generate random variation in the input link lengths and obtain the deviation response from the fitted Response Surface equations. From this deviation, the failure probability and the reliability for the corresponding crank angle is calculated. The total retraction time for the Nose landing gear is 10 seconds. The response angle (ψ) has been evaluated at intervals of 1 second as given in Table-1.

Table-1. Response measure for the complete retraction cycle.

Retraction Time in Seconds	Trial	Response Measured in Degrees	Clearance Level in m	Ideal Value	Error
1	1	194.58	0		0.1749
	2	194.12	0.0015	194.405	-0.2851
	3	193.84	0.003		-0.5651
2	1	210.28	0.0000		-0.0118
	2	209.95	0.0015	210.291	-0.3418
	3	209.63	0.003		-0.6618
3	1	228.61	0		0.0685
	2	228.02	0.0015	228.541	-0.5215
	3	227.6	0.003		-0.9415

The input parameters for both the Impact Function and Coefficient of Restitution Model, Viz: Four Input parameters (Stiffness, Force Exponent, Damping and the penetration Depth) in case of Impact Function Model and two input parameters (Penalty and Restitution Coefficient) in case of Coefficient of Restitution Model has been obtained by an optimization process using inbuilt Design of Experiments tool available in MSC ADAMS, in absence of the field data for the same.

MONTE CARLO SIMULATION

The Monte Carlo simulation is a technique by which its distribution and statistical characteristics can be approximately calculated through sampling the random source of errors with any distribution and then simulating the stochastic model of the kinematic error. This technique eliminates the necessity of complicated probability calculations [4]. In case of the present study, the variations in joint clearance have been assumed to be normally distributed with mean μ and standard variation σ. Thus for any Joint Clearance 'v' the change Δv can be modelled as:

$$\Delta v = f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}} \quad (12)$$

The mean value of the Joint Clearance has been assumed to be C = 1.5 mm and the standard deviation used for the random variable input for Monte Carlo simulation is σ = 0.3947 mm. A mechanism can be represented by the equation

$$F(U, V, R) = 0 \quad (13)$$

Where R is the mechanism structural parameters, U is the motion output parameters and V is the motion input parameters. If ΔV and ΔR are the errors in the input and mechanism structural parameters, the error in the motion output parameter ΔU can be computed by using the relation,

$$F(U + \Delta U, V + \Delta V, R + \Delta R) = 0 \quad (14)$$

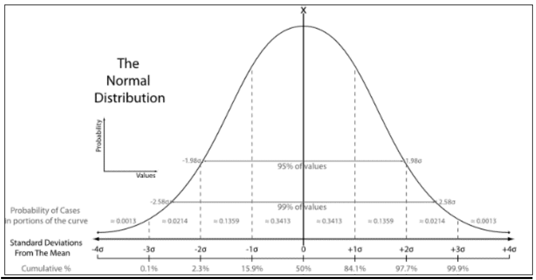

Figure-4. The normal distribution for variation in joint clearance [4].

A limit for the output parameter ΔU is chosen based on the required accuracy of the mechanism, which in the present case is 1.85^0.and the number of times the limit is exceeded is counted for the random variation of the mechanism structural parameters for a large number of iterations. The ratio of the number of times the limit is exceeded to the number of iterations gives the unreliability for the mechanism and vice-versa. In the present study, the number of iterations arrived for the mechanism structural parameters after convergence studies has been 100000. A large number of iterations are necessary for completely capturing the stochastic variation of the motion of the structural parameters. The present study has been restricted for the variation in Joint Clearance.

RESULTS AND DISCUSSION

The Impact function model has four parameters (Stiffness, Force Exponent, Damping and Penetration Depth).In the absence of practical data for the above mentioned parameters, a design of experiments (DOE) based parameter optimization has been performed in MSC

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ADAMS with the objective to minimize the error, when compared with the ideal joint.A three level full factorial DOE has been performed to obtain the optimized value. The nominal values and the limits have been shown in Table-2.

Table-2. Input parameters for impact function model parameter optimization.

Variables	Nominal Value	Absolute Upper/Lower Limit
Stiffness in N/m	$1.1e^{+008}$	$1.0e^{+008}/1.2004e^{+008}$
Force Exponent	1.44	1.2960/1.5840
Damping in N.s/m	$2.0e^{+4}$	$1.0e^{+4}/3.0e^{+4}$
Penetration Depth in m	0.0001	$9.9e^{-5}/0.000101$

The Optimized value for the Impact function model parameters have been computed using MSC ADAMS INSIGHT and the values were found to be:

Stiffness in N/m	:	$1.2004\ e^{+008}$
Force Exponent	:	1.2960
Damping in N.s/m	:	$1\ e^{+4}$
Penetration Depth in m	:	$9.9\ e^{-5}$

The Optimized Impact Function Model comparison is shown in Table-3.

Table-3. Impact function model optimization.

Retraction Time	Ideal Value	Initial Value	Optimized Value	Initial Error	Reduced Error
1	194.4051	194.36	194.58	0.0451	0.1749
2	210.2918	210.2	210.28	0.0918	0.0118
3	228.5415	228.26	228.61	0.2815	0.0685
4	250.2004	249.57	250.1200	0.6304	0.0804
5	275.7497	274.48	275.52	1.2697	0.2297
6	302.9595	300.77	302.46	2.1895	0.4995
7	**324.8278**	**322.29**	**324.2**	**2.5378**	**0.6278**
8	336.5902	334.65	336.14	1.9402	0.4502
9	341.5261	340.26	341.24	1.2661	0.2861
10	343.3430	342.48	343.1800	0.8630	0.1630

The restitution model has two parameters (Penalty and Coefficient of Restitution). A three level full factorial design of Experiments have been performed to obtain the optimized Value of the parameters in the same way as explained for the impact model approach. The nominal values and the limits has been shown in Table-4.

Table-4. Input parameters for restitution model parameter optimization.

Variables	Nominal Value	Absolute Upper/Lower Limit
Penalty	$1.0e^{+008}$	$9.99e^{+007}/1.0010e^{+008}$
Restitution Coefficient	0.5	0.3/0.7

Table-5. shows that the highest error occurs at a retraction time of (t) = 8 seconds, for which the optimization has been carried out.

Table-5. Restitution model optimization.

Retraction Time	Ideal Value	Initial Value	Optimized Value	Initial Error	Reduced Error
1	194.405	194.429	194.45	0.024	0.0449
2	210.291	210.199	210.31	0.091	0.0182
3	228.541	228.486	228.56	0.055	0.0185
4	250.200	250.082	250.23	0.118	0.0296
5	275.749	275.521	275.74	0.228	0.0097
6	302.959	302.568	302.71	0.391	0.2495
7	324.827	324.272	324.43	0.555	0.3978
8	**336.590**	**336.004**	**336.14**	**0.585**	**0.4502**
9	341.526	341.002	341.11	0.523	0.4161
10	343.343	342.891	342.97	0.452	0.3730

The Optimized values of the parameters for the restitution based approach based model have been evaluated using MSC ADAMS INSIGHT and the values were found to be:

Penalty	:	$1.0e^{+8}$
Coefficient of Restitution	:	0.5

The restitution model is extremely sensitive to the duration of the contact event, and is best suited for impulse type simulations. It is not ideal for time histories that include continuous contact events [4]. Response surface coefficients have been computed using ADAMS Insight and the reliability for the mechanism at different retraction times has been computed using Monte Carlo simulations using in-house codes created using MATLAB software. Table-6. shows the comparison between Impact, restitution and clearance models.

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Table-6. Reliability comparison between impact, restitution and clearance link models.

Retraction Time	Crank Angle	Impact	Restitution	Clearance Link
		Dev=1.85	Dev=1.85	Dev=1.85
1	40.262	1	1	1
2	36.506	1	1	1
3	32.972	1	1	1
4	29.621	1	1	1
5	26.422	1	1	1
6	23.353	0.960	0.777	1
7	20.397	0.889	0.475	1
8	17.538	0.998	0.847	1
9	14.765	1	1	1
10	12.068	1	1	1

The Clearance Link Model shows the highest reliability in comparison with the Impact and Restitution Model, but it should be noted that it ignores friction, stiffness and damping effects and thus is not realistic and does not render accurate results.

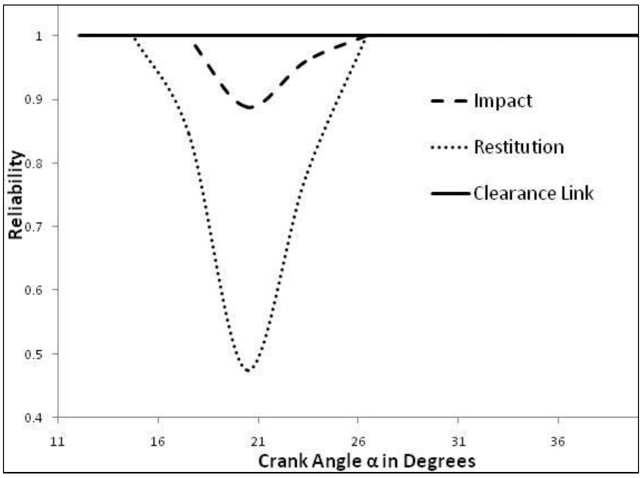

Figure-5. Reliability plot comparison between impact, restitution and clearance link models.

The results obtained from the response surface method based Monte Carlo simulation has been compared with the Direct Monte Carlo Simulation at 7 and 8 seconds of the retraction process as shown in Table-7.

Table-7. Reliability comparison between RSM based MCS and direct MCS for impact function model (10000 iterations).

Retraction Time (t) in Seconds	RSM based Monte Carlo Simulation	Direct Monte Carlo Simulation
7	0.8891	0.8712
8	0.9988	0.9813

A random normal distribution based input of the Joint clearance produces random responses from which the errors have been evaluated to compute the reliability. As complete simulations in ADAMS have to be carried out for all the 10000 iterations, the computational time required is higher. In contrast, in case of the response surface based MCS, the computational time required is less. The Reliability values shown in the above table are in close conformance with each other, which validates that the response surface bases MCS approach can give results with reasonable accuracy to evaluate the reliability of Joint Clearance.

CONCLUSIONS

The Clearance Link Model being an approximate approach is suitable for kinematics but leads to increased errors in case of dynamics, where in friction, stiffness, and damping is involved. The restitution Model is ideal for impulse simulations where continuous contact does not exist. Impact Function model has found to be the ideal choice for realistic applications. The input parameters for the impact modelling approach, if not available can be obtained by a Design of Experiments approach for the parameters and optimizing it such that it gives the least error in the response value when compared to the mechanism for no clearance condition. The lowest reliability of the retraction mechanism occurs at a retraction time, t= 7 seconds, which coincides with the maximum actuator force. Reliability increases with the decrease in the joint clearances. The results obtained from Response Surface based Monte Carlo and Direct Monte Carlo simulations have been found to be in good agreement with each other. The studies have revealed the importance of choice of the proper contact model and the corresponding model parameters in the results obtained.

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